# CAUSAL BULK VISCOUS LRS BIANCH I MODELS WITH VARIABLE GRAVITATIONAL AND COSMOLOGICAL "CONSTANT"

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## Abstract

In this paper we have investigated an LRS Bianchi I anisotropic cosmological model of the universe by taking time varying $G$ and $\Lambda$ in the presence of bulk viscous fluid source described by full causal non-equilibrium thermodynamics. We obtain a cosmological constant as a decreasing function of time and for $m, n > 0$, the value of cosmological "constant" for this model is found to be small and positive which is supported by the results from recent supernovae observations.

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# 1　Introduction

The conventional theory of evolution of the universe includes a number of dissipative processes. Dissipative thermodynamics processes in cosmology originating from a bulk viscosity are believed to play an important role in the dynamics and evolution of the universe. Misner[1] suggested that large-scale isotropy of the universe observed at the present epoch is due to action of neutrino viscosity which was not negligible when the universe was less than a second old. A number of processes responsible for producing bulk viscosity in the very early universe are such as the interaction between radiation and matter[2], gravitational string production [3, 4], viscosity due to quark and gluon plasma, dark matter or particle creation[5, 6]. It is important that each dissipative process is subject to as detailed analysis as possible. However, it is also important to develop a robust model of dissipative cosmological processes in general, so that one can analyze the overall dynamics of dissipative without getting lost in the details of particular complex processes. In requirements of such a model Maartens[7] pointed out that the model should (i) be causal and stable, and (ii) provide a constant relativistic thermodynamics in the 'conventional' post-inflationary regime.


In order to study these phenomena, the theory of dissipative was first developed by Eckart[8] and subsequently an essential equivalent formulation was given by Landau and Lifshitz[9]. But Eckart theory has several drawbacks including violation of causality and instabilities of equilibrium states. Readers interested in the general theory of causal thermodynamics are urged to consult the excellent survey report of Maartens[7] and Zimdahl[10] and references cited therein. A relativistic second-order theory was found by Israel[11] and developed by Israel and Stewart[12]. The advantages of the causal theory are as follows[13]: (1) For stable fluid configurations, the dissipative signals propagate causally. (2) Unlike Eckart-type's theory, there is no generic short- wavelength secular instability in causal theory. (3) Even for rotating fluids, the perturbations have a well-posed initial value problem. Therefore, the best currently available theory for analyzing dissipative processes in the universe is the full (i.e. non-truncated) Israel-Stewart causal thermodynamics, which we consider in this work.


In recent years, models with relic cosmological constant $\Lambda$ have drawn considerable attention among researchers for various aspects such as the age problem, classical tests, observational constraints on $\Lambda$, structure formation and gravitational lenses have been discussed in the literature (see Refs.[14]−[16]). Lindey[17] has suggested that cosmological "constant" may be considered as a function of temperature and related to the spontaneous symmetry breaking process. Therefore, $\Lambda$ should be a function of time in a homogeneous universe as temperature varies with time. Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmo-

logical constant are given by Ratra and Peebles [18], Dolgov[19]−[21], Sahni and Starobinsky[22], Peebles[23], Padmanabhan[24] and Vishwakarma[25]. Recent cosmological observations suggest the existence of a positive cosmological constant $\Lambda$ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. There are several aspects of the cosmological constant both from cosmological and field theoretical perspectives. Presently, determination of $\Lambda$ has become one of the main issues of modern cosmology as it provides the gravity vacuum state and make possible to understand the mechanism which led the early universe to the large scale structures and to predict the fate of the whole universe. The cosmological "constant" can be measured by observing quasars whose light gets distorted by gravity of galaxies that lies between the quasars and Earth. Krauss and Turner[26] have mentioned that as $\Lambda$ term dominates the energy density of the universe, cosmologists are correct in their attempt to evoke it once again for better understanding of both the universe and fundamental physics.

In the last few decades there have been numerous modifications of general relativity in which gravitational "constant" $(G)$ varies with time[27]. Considering the principle of absolute quark confinement, Der Sarkissian[28] has suggested that gravitational and cosmological "constant" may be considered as functions of time parameter in Einstein's theory of relativity. A number of authors[29]−[40] have considered time-varying $G$ and $\Lambda$ within the framework of general relativity.

Motivated by the fact that bulk viscosity, gravitational and cosmological "constants" , are more relevant during early stages of the universe, in this paper, we have considered the evolution of a LRS Bianchi I model with bulk viscous fluid in full causal non-equilibrium thermodynamics, in presence of time-varying gravitational and cosmological "constants".

## 2   THE Basic Equations

A locally rotationally symmetric (LRS) Bianchi I space-time may be represented by the line element

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2),  \tag{1}$$

where metric potentials A and B are depending on cosmic time $t$ only.
The Einstein's field equations with variable $G$ and $\Lambda$ are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda(t)g_{ij} = -8\pi G(t)T_{ij},  \tag{2}$$

where $R_{ij}$ is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar; and $T_{ij}$ is the energy-momentum tensor of cosmic fluid in the presence of bulk viscosity given

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by
$$T_{ij} = (\rho + p + \Pi)u_i u_j - p g_{ij}. \tag{3}$$

Here $\rho$, $p$ and $\Pi$ are the energy density, equilibrium pressure and bulk viscous pressure respectively and $u^i$ is the flow vector satisfying the relations $u^i u_i = 1$. The Einstein's field equations (2) for the line element (1) lead to the following set of equations

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi G(t)\rho + \Lambda(t), \tag{4}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = -8\pi G(t)[p + \Pi] + \Lambda(t), \tag{5}$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -8\pi G(t)[p + \Pi] + \Lambda(t). \tag{6}$$

A combination of equations (4) - (6) yield the continuity equation

$$8\pi \dot{G}\rho + 8\pi G\left[\dot{\rho} + (\rho + p + \Pi)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right)\right] + \dot{\Lambda} = 0. \tag{7}$$

The usual energy-momentum conservation equation $T^{ij}_{;j} = 0$ suggests

$$\dot{\rho} + (\rho + p + \Pi)\left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right] = 0. \tag{8}$$

From equations (7) and (8), we get

$$\dot{\Lambda} = -8\pi \dot{G}\rho. \tag{9}$$

For the full causal non-equilibrium thermodynamics, the causal evolution equation for bulk viscosity is given by[7]

$$\tau\dot{\Pi} + \Pi = -\xi\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) - \frac{\epsilon}{2}\,\tau\,\Pi\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right), \tag{10}$$

where $T \geq 0$ is the temperature, $\xi$ the bulk viscous coefficient and $\tau \geq 0$ the relaxation coefficient for the transient bulk viscous effect (relaxation time i.e. the time which system takes in going back to equilibrium once the divergence of the four velocity has been switched off). For $\tau = 0$, equation (10) gives evolution equation for the non-causal theory. For $\epsilon = 0$, we get causal evolution equation for truncated theory, which implies a drastic condition on the temperature, while $\epsilon$ takes value unity in full causal theory.

# 3 The Model

Equations (5) and (6) yield

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} = 0. \tag{11}$$

In order to find exact solutions of the field equations, we require more physically plausible relations amongst the variables. Considering a power law relation between $A$ and $B$ as $A \sim B^n$, eq.(11) suggests

$$B = B_0 \left(\frac{t}{t_0}\right)^{\frac{1}{n+2}}, \tag{12}$$

$$A = A_0 \left(\frac{t}{t_0}\right)^{\frac{n}{n+2}}, \tag{13}$$

where $A_0$ and $B_0$ are the values of $A$ and $B$ at present time $t = t_0$ and $n$ is a positive constant.

By use of (12) and (13), equations (4) - (6), reduce to

$$\frac{2n}{(n+2)^2} \frac{1}{t^2} = 8\pi G\rho + \Lambda, \tag{14}$$

$$\frac{2n+1}{(n+2)^2} \frac{1}{t^2} = 8\pi G(p + \Pi) - \Lambda. \tag{15}$$

Further following[21]−[25], we assume $G = t^m$. Hence equations (9) and (14) suggest

$$\dot{\Lambda} - \frac{m}{t}\Lambda = -\frac{2mn}{(n+2)^2} \frac{1}{t^3}, \tag{16}$$

which is a linear equation and it has solution

$$\Lambda = \frac{2mn}{(n+2)^2(m+2)} \frac{1}{t^2}. \tag{17}$$

From equations (14) and (17) one can easily obtain expression for energy density in terms of cosmic time $t$ as

$$\rho = \frac{n}{2\pi(m+2)(n+2)^2} \frac{1}{t^{m+2}}. \tag{18}$$

Considering the usual barotropic equation of state relating the perfect fluid pressure $p$ to the energy density as

$$p = (\gamma - 1)\rho, \tag{19}$$

where $\gamma(1 \leq \gamma \leq 2)$ is a constant, equations (13) and (14) lead to

$$\Pi = \frac{n(m+1)}{2\pi(m+2)(n+2)^2} \frac{1}{t^{2+m}}. \tag{20}$$

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Now, we consider[7, 10][41]−[44] following phenomenological widely accepted relations

$$\xi - \xi_0 \rho^\alpha \ \ and \ \ \tau = \frac{\xi}{\rho} \tag{21}$$

for the bulk viscosity coefficient $\xi$ and mass density $\rho$ and also for bulk viscosity coefficient and the relaxation time $\tau$, respectively, where $\xi_0 \geq 0$ and $\alpha$ are constants. If $\alpha = 1$, eq.(21) may correspond to a radiative fluid, whereas $\alpha = \frac{3}{2}$ may correspond to a string-dominated universe. However, more realistic models are based upon $\alpha$ in the region $0 \leq \alpha \leq \frac{1}{2}$.
Using (21), equation (10) on integration yields

$$T = T_0 \ exp \left[\frac{2}{\epsilon} g(t)\right] \ exp \left[\frac{2}{\epsilon} f(t)\right] \frac{\Pi^{\frac{2}{\epsilon}} R^3 \tau}{\xi}, \tag{22}$$

where $f(t)$ and $g(t)$ are anti-derivatives of $\frac{1}{\tau}$ and $\frac{3\rho H}{\Pi}$ respectively.
With the help of equations (18), (20) and (21), we obtain the expressions for $f(t)$ and $g(t)$ as

$$f(t) = \frac{\tau_0}{t^{(m+2)(\alpha-1)-1}}, \tag{23}$$

$$g(t) = \frac{1}{m(m+1)t^m}, \tag{24}$$

where

$$\tau_o = \frac{1}{1 - (m+2)(\alpha-1)} \left[\frac{n}{2\pi(m+2)(n+2)^2}\right]^{\alpha-1}. \tag{25}$$

We observe from eq. (17) that the cosmological constant in this model is decreasing function of time and it approach a small value as time increases ( i.e., the present epoch). For $m, n > 0$ the value of cosmological "constant" for this model is found to be small and positive which is supported by the results from recent supernovae observations recently obtained by the High - z Supernova Team and Supernova Cosmological Project ( Garnavich *et al.*;[45] Perlmutter *et al.*;[46] Riess *et al.*;[47] Schmidt *et al.* [48]).
Using the observational values $\dot{G}/G = 10^{-11} yr^{-1}$ and $H_p = 7.5 \times 10^{-11} yr^{-1}$, and the relation of the present age of the universe with Hubble parameter $(H_p \ t_p \sim \frac{2}{3})$, relation $G = t^m$ suggest

$$m = \frac{2}{22.5}$$

This clearly shows that gravitational parameter $G$ turns out to be an increasing function of time.
From eq (18), it is observed that the energy density $\rho$ is decreasing with evolution of the universe. For all $m > -2$, $n > 0$, the energy conditions are satisfied.
It is also observed from eq (20) that bulk viscous pressure $\Pi$ decreases with time. When $t \to 0$, $\Pi \to \infty$. When $t \to \infty$, $\Pi \to 0$.
In this model, expressions for expansion and shear are

$$\theta = \frac{1}{t} \ \ and \ \ \sigma^2 = \frac{2(n-1)^2}{3(n+2)^2 t^2}. \tag{26}$$

$$\frac{\sigma^2}{\rho} = \frac{4\pi(n-1)^2(m+2)}{3n}t^m.$$ 

(27)

Equation (27) clearly shows the effect of time varying $G$ on the relative anisotropic. For, $-2 < m < 0$, relative anisotropy is decreasing. Further, it can be observed that $\sigma^2 \propto \theta^2$, which indicate that the model does not approach isotropy for large value of $t$.

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